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Extending the QCD Perturbative Domain to Higher Energies

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After a brief introduction to low-x QCD and to resummation approaches, I illustrate the predictions for the gluon Green function and splitting function of a recent renormalization–group improved small-x resummation scheme. I argue, on this basis, that the range of validity of perturbative calculations is considerably extended in rapidity with respect to leading log expectations. The perturbative high-energy exponents are predicted in a phenomenologically interesting range, and significant preasymptotic effects are found. In particular, the splitting function shows a shallow dip in the moderate-x region, followed by the expected small-x power increase.

1 Low-x Physics in QCD

The description of high-energy hard cross–sections in QCD perturbation theory has a long history. Starting from the early BFKL prediction of rising cross sections [1] at leading logs level (LL) and from the k-factorization framework [2], up to the difficult calculation of the next–to–leading (NLL) kernel [3, 4] and to its improvements [5, 6, 7, 8, 9, 10, 11, 12], a lot of work has been devoted to fill the gap between theory and small-x phenomenology.

On the experimental side, the DIS structure functions at HERA [13] show a rise which is well described by large scaling violations arising in low order DGLAP [14] evolution. On the other hand, data for two–scale processes like $\gamma^*(Q)-\gamma^*(Q_0)$ for which the DGLAP picture is inappropriate, seem incompatible with the strong rise predicted by LL BFKL evolution. There is thus a need to reconcile the two theoretical pictures, which actually differ by the order of resummation of large logarithms (log Q^2 vs. log s) in the perturbative series.

On the theoretical side, one expects QCD perturbation theory at low x to be governed by some effective coupling $\alpha_s(\mathbf{k}^2)\log 1/x$ which risks to be strong for two independent reasons. Firstly, because of the logarithmic enhancements due to the opening of the small$x$ gluon phase space. This motivates the calculation of next-to-leading corrections to the hard-Pomeron intercept, which however turn out to be quite large and negative [3, 4], pointing to some instability of the log s hierarchy. Secondly, because energy evolution – by diffusion or tunneling [15] – may give rise to low values of $\mathbf{k}$, the gluon momentum transfer, thus implying a drift towards a really non-perturbative asymptotic regime [16, 17, 18, 19], characterized by soft physics. We must control, therefore, higher order subleading terms, and analyze the perturbatively accessible rapidity range.

A tentative answer to the above question marks about the perturbative behaviour comes from the (double) resummation approaches. The idea is to take into account higher order subleading terms in both variables (log s and log Q^2), based on some distinctive features of such terms. For instance, the "duality" approach [8, 9] starts from the approximate symmetry between scaling violations and energy dependence in order to hint at a resummed form of the gluon splitting function. The "renormalization-group improved" (RGI) approach [5, 6, 7] starts from BFKL evolution at NLL level, and resums those higher order subleading terms which are required by the RG behaviour for both $Q \gg Q_0$ and $Q_0 \gg Q$. Other resummation models are available also [10, 11, 12]. The RGI approach is able to provide predictions [20] for both the gluon splitting function and the gluon density itself, and I will concentrate on that approach in the following.

The BFKL equation is, to start with, an evolution equation in the rapidity $Y = \log(s/QQ_0)$, induced by a kernel $K(\mathbf{k},\mathbf{k}')$ in $\mathbf{k}$ –space, which can be expanded in α_s and has been calculated up to second order. The LL level is dominated by (effective) ladder diagrams with gluon exchange and the corresponding kernel is scale–invariant and given by ($\bar\alpha_s = N_c\alpha_s/\pi$).

$$\alpha_s K_0(\mathbf{k},\mathbf{k}') = \frac{\bar\alpha_s}{(\mathbf{k}-\mathbf{k}')^2} \; + \; \textit{virtual terms}. \qquad (1)$$

The highest eigenvalue of K_0 yields the exponent for the energy dependence and is $\omega_s = 4\log 2\bar\alpha_s$, a value which is too large compared to HERA data (e. g., 0.55 for α_s=0.2, compared to an observed value of about 0.2). Similarly, the gluon anomalous dimension, given by $\bar\alpha_s/\omega$ in $\omega = N-1$ moment space at one–loop level, acquires higher order singularities with positive residues. Roughly speaking, the LL approximation grossly overestimates both two–scale cross–sections' rise and scaling violations.

The NLL kernel, extracted in [3, 4] after many years of work of various groups, yields some qualitatively new features and question marks. Firstly, the corrections to the high–energy exponent are negative (which is good), but with a quite large coefficient, as follows

$$\omega_s = \omega_s^0\ (1 - 6.47\ \bar\alpha_s\ +\ ...). \qquad (2)$$

This points towards an instability of the leading–log s hierarchy, which is confirmed by the risk of oscillatory behaviour of the gluon density [21] for sizeably different external scales. Secondly, running coupling effects are taken into account, and the scale of the coupling suggested by the kernel is $(\mathbf{k}-\mathbf{k}')^2 = q^2$. Therefore, in order to avoid divergent integrations, we need to regularize the Landau pole, by introducing some cutoff (or freezing parameter) $\bar{k}$. However, when the energy increases, there is a favoured evolution towards smaller values of $\mathbf{k} = \mathcal{O}(\bar{k})$, by diffusion or tunneling, and this implies that the asymptotic high–energy exponent becomes $\bar{k}$-dependent, i.e., non–perturbative (asymptotic Pomeron). Therefore, NLL calculations raise more questions than they are able to answer, and call for the inclusion of higher order terms.

2 The Renormalization Group Improved Approach

Higher order subleading contributions are not known in detail. We can argue, however, that a class of them are large, and required in order to build single–logarithmic scaling violations when either Q or Q_0 is the leading scale [5], and Q^2/s or Q_0^2/s is the corresponding Bjorken variable. Taking into account such terms is equivalent to resumming subleading terms in log s which are known because they are leading in log Q^2 and are provided therefore by the renormalization group. This idea was used in order to build the RGI approach of [7], where the solution of the homogeneous small-x equation was studied in detail by the use of the ω–expansion method [6] so as to calculate stable high–energy exponents and gluon anomalous dimension. In recent papers [20] the approach has been extended to the full inhomogeneous equation in a slightly different resummation scheme, which is more suitable for numerical evaluation. I am thus able to describe results for both the two–scale gluon density and for the splitting function in the collinear limit. Results for the splitting function have been recently obtained in the duality approach also [22].

The basic problem considered in [20] is the calculation of the (azimuthally averaged) gluon Green function $G(Y;k,k_0)$ as a function of the magnitudes of the external gluon transverse momenta $k \equiv |\mathbf{k}|$, $k_0 \equiv |\mathbf{k}_0|$ and of the rapidity $Y \equiv \log \frac{s}{kk_0}$. This is not yet a hard cross section, because we need to incorporate the impact factors of the probes [2, 23, 24]. Nevertheless, the Green function exhibits most of the physical features of the hard process, if we think of k^2, k_0^2 as external (hard) scales. The limits $k^2 \gg k_0^2$ ($k_0^2 \gg k^2$) correspond conventionally to the ordered (anti-ordered) collinear limit. By definition, in the ω-space conjugate to Y (so that $\hat{\omega} = \partial_Y$) we set

$$\mathcal{G}_\omega(\mathbf{k},\mathbf{k}_0) \equiv [\omega - \mathcal{K}_\omega]^{-1}(\mathbf{k},\mathbf{k}_0)\,, \qquad (3)$$

and

$$\omega \mathcal{G}_\omega(\mathbf{k},\mathbf{k}_0) = \delta^2(\mathbf{k}-\mathbf{k}_0) + \int d^2\mathbf{k}'\, \mathcal{K}_\omega(\mathbf{k},\mathbf{k}')\mathcal{G}_\omega(\mathbf{k}',\mathbf{k}_0)\,, \qquad (4)$$

where $\mathcal{K}_\omega(\mathbf{k},\mathbf{k}')$ is a kernel to be defined, whose $\omega = 0$ limit is related to the BFKL Y-evolution kernel discussed before.

The precise form of the kernel $\mathcal{K}_\omega$ is given in Ref. [20] However, the basic features of the RGI approach is illustrated by a simple observation: in BFKL iteration, all possible orderings of transverse momenta are to be included, the ordered (anti-ordered) sequence $k \gg k_1 \cdots \gg k_n \cdots \gg k_0$ ($k \ll k_1 \cdots \ll k_n \cdots \ll k_0$) showing scaling violations with Bjorken variable k^2/s (k_0^2/s). Therefore, if only leading $\log k^2$ contributions were to be considered, the kernel $\mathcal{K}_\omega$ acting on $\frac{1}{k^2}\left(\frac{k^2}{k_0^2}\right)^\gamma$ would be approximately represented by the following eigenvalue function (in the frozen coupling limit)

$$\frac{1}{\omega}\mathcal{K}_\omega \to \bar{\alpha}_s \left(\frac{1}{\gamma+\frac{\omega}{2}} + \frac{1}{1+\frac{\omega}{2}-\gamma}\right)\left(\frac{1}{\omega} + A_1(\omega)\right) + \cdots$$
$$\bar{\alpha}_s \equiv \alpha_s \frac{N_c}{\pi}\,, \qquad (5)$$

where $\gamma_{gg}^{(1)} = \bar{\alpha}_s\left(\frac{1}{\omega} + A_1(\omega)\right)$ is the one-loop gluon-gluon anomalous dimension and we have introduced the variable γ conjugate to $\log k^2$. Note, in fact, that Eq. (5) reduces to the normal DGLAP evolution [14] in $\log k^2$ ($\log k_0^2$) in the two orderings mentioned before, because $\gamma + \frac{\omega}{2}$ ($1 + \frac{\omega}{2} - \gamma$) is represented by $\partial_{\log k^2}$ ($\partial_{\log k_0^2}$) at fixed values of $x = k^2/s$ ($x_0 = k_0^2/s$) in the ordered (anti-ordered) momentum region. Note also the ω-dependent shift [5, 6, 7] of the γ-singularities occurring in Eq. (5), which is required by the change of scale (k_0^2 versus k^2) needed to interchange the orderings, i.e., x_0 versus x.

We thus understand that the ω-dependence of $\mathcal{K}_\omega$ is essential for the resummation of the collinear terms and can be used to incorporate the exact LL collinear behaviour, while on the other hand, the $\omega \to 0$ behaviour of $\mathcal{K}_\omega$ is fixed by the BFKL limit up to $\mathcal{O}(\omega)$ terms, so as to incorporate exact LL and NLL kernels. Such requirements fix the kernel up to contributions that are NNL in $\ln x$ and NL in $\ln Q^2$. The resulting integral equation to be solved by the definition (4) is thus a running coupling equation with non linear dependence on α_s at appropriate scales, and it has a somewhat involved ω-dependence in the improved kernels $\mathcal{K}_\omega$. Its solution has been found in [20] by numerical matrix evolution methods in k- and x- space [25], where the typical ω-shifted form in the example (5) corresponds to the so-called consistency constraint [26, 27, 28]. Furthermore, introducing the integrated gluon density

$$xg(x,Q^2) \equiv \int^{Q^2} d^2\mathbf{k}\, G^{(s_0=k^2)}(\log 1/x; |\mathbf{k}|, k_0)\,, \qquad (6)$$

$$\mathscr{G}_\omega^{(s_0=k^2)} \equiv \left(\frac{k}{k_0}\right)^\omega \mathscr{G}_\omega \,, \tag{7}$$

the resummed splitting function $P_{\rm eff}(z,Q^2)$ is defined by the evolution equation

$$\frac{\partial g(x,Q^2)}{\partial \log Q^2} = \int \frac{dz}{z}\, P_{\rm eff}(z,Q^2)\, g\left(\frac{x}{z},Q^2\right) , \tag{8}$$

and has been extracted [20] by a numerical deconvolution method [29]. We note immediately that $P_{\rm eff}$ turns out to be independent of k_0 for $Q^2 \gg k_0^2$, yielding an important check of RG factorisation in our approach.

3 Gluon Green's Function

Results for $G(Y;k,k)$[1] are shown in Fig. 1. In addition to the solution based on our RGI approach ($\mathrm{NLL_B}$) the figure also has 'reference' results for LL evolution with kernel $\bar\alpha_s(x_\mu^2 q^2)K_0^0$. The one-loop running coupling (with $n_f = 4$), is regularised either by setting it to zero below a scale $\bar k$ ('cutoff') or by freezing it below that scale ($\alpha_s(q^2 < \bar k^2) = \alpha_s(\bar k^2)$). The cutoff regularisation is supposed to be more physical since it prevents diffusion to arbitrarily small scales and is thus more consistent with confinement – accordingly three cutoff regularisations are shown and only one frozen regularisation. The $\bar k = 0.74$ GeV cutoff solution is presented together with an uncertainty band associated with the variation of x_μ^2 between $\frac{1}{2}$ and 2, x_μ being a renormalization–scale testing parameter, such that $\bar\alpha_s(q^2)$ is replaced by $\bar\alpha_s(x_\mu^2 q^2) + b\ \bar\alpha_s^2\ log\ x_\mu^2$ in the part of the kernel linear in α_s.

Solutions of (4) with an IR-regularised coupling generally have two domains [16, 17, 18, 15], separated by a critical rapidity $Y_c(k^2)$. For the intermediate high-energy region $1 \ll Y < Y_c(k^2)$, one expects the perturbative 'hard Pomeron' behaviour with exponent ω_s,

$$k^2 G(Y;k,k) \sim \frac{1}{\sqrt{Y}} \exp\left[\omega_s(\alpha_s(k^2))Y + \Delta(\alpha_s,Y)\right] , \tag{9}$$

and diffusion corrections [30, 31, 32, 33] parametrised by $\Delta(\alpha_s,Y)$. Beyond Y_c, a regularisation-dependent non-perturbative 'Pomeron' regime takes over

$$k^2 G(Y;k,k) \sim \left(\frac{\bar k^2}{k^2}\right)^\xi e^{\omega_P Y} , \qquad \begin{array}{rl} \mathrm{LL}: & \xi = 1 \\ \mathrm{NLL_B}: & \xi = 1+\omega_P \end{array} \tag{10}$$

where the non-perturbative exponent ω_P satisfies [7] $\omega_P \sim \omega_s(\alpha_s(\bar k^2))$ and hence is formally larger than $\omega_s(\alpha_s(k^2))$.[2]

[1] Actually,slightly different values of the scales in G are taken, namely $G(Y;k+\varepsilon,k-\varepsilon)$ with $\varepsilon = 0.1k$, in order to avoid sensitivity to the discretisation of the δ-function initial condition in 4 (cf. Ref. [20] for a detailed discussion).

[2] The behaviour 10 with $\omega_P > \omega_s(\alpha_s(k^2))$ is a general feature of linear evolution equations such as 4, but not of actual high energy cross sections, which are additionally subject to non-linear effects and confinement.

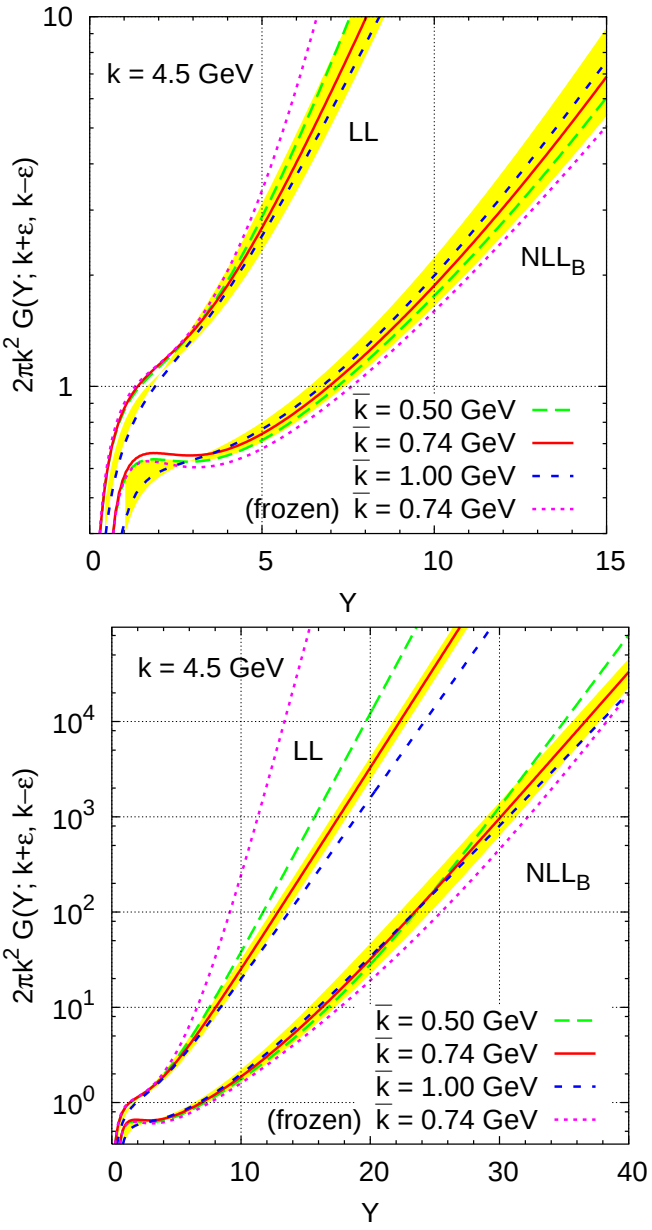

Figure 1. *Green's function calculated with four different infrared regularisations of the coupling, shown for LL and RGI NLL ('$\mathrm{NLL_B}$') evolution. The bands indicate the sensitivity of the $\bar k = 0.74$ GeV results to a variation of x_μ^2 in the range $\frac{1}{2}$ to 2. The left and right hand plots differ only in their scales.*

The value of Y_c depends strongly on k. In the tunnelling approximation, it can be roughly estimated by equating eqs. (9) and (10) to yield [34, 15], for any given regularisation procedure,

$$Y_c(k^2) \simeq \frac{\xi \log(k^2/\bar k^2)}{\omega_P - \omega_s(\alpha_s(k^2))} , \tag{11}$$

(again with $1+\omega_P \to 1$ for LL), showing an approximately linear increase of Y_c with $\log k^2$.

Within this logic, several aspects of Fig. 1 are worth commenting. The most striking feature of the LL evolution is its strong dependence on the non-perturbative regularisation, even for rapidities as low as 5. The exact value of Y_c depends on the regularisation being used, ranging between 5 and 10. In contrast, $\mathrm{NLL_B}$ evolution remains under perturbative control up to much larger rapidities and the NP pomeron behaviour takes over only for $Y > 25$, where the three cutoff solutions start to diverge[3]. Therefore, Y_c is considerably larger for the resummed evolution, as consequence of the fact that subleading corrections lower both the PT and – even more – NP exponents (see Eq.11).

We note that at their respective Y_c's the $\mathrm{NLL_B}$ Green's function is an order of magnitude larger than the LL one: the subleading corrections increase the overall amount of BFKL growth remaining within perturbative control. However, large densities $G \sim 1/\alpha_s$ are reached at cosiderably larger values of Y, so that saturation effects [30] are pushed towards higher energies. For the reference values $k = 4.5$ GeV, $\alpha_s \sim 0.2$ this translates to Y of order 15, close to the kinematical limit of LHC, as a very rough estimate.

In Fig. 1 only a single value of k is considered. The question of NP contributions is summarised more generally in Fig. 2, which shows contour plots of the logarithmic spread of the four regularisations. Darker regions are less IR sensitive, and contours for particular values of the spread have been added to guide the eye. Here too one clearly sees the much larger region (including most of the phenomenologically interesting domain) that is accessible perturbatively after accounting for subleading corrections.

So let us now therefore return to Fig. 1 and examine the characteristics of the $\mathrm{NLL_B}$ Green's function in the perturbatively accessible domain, which should be describable by an equation of the form (9). The first feature to note is that the growth starts only from $Y \sim 4$. This suggests that at today's collider energies (implying $Y < 6$ [35, 36]), it will at best be possible to see only the start of any growth. This preasymptotic feature is partly due to the slow opening of small-x phase space [37] implicit in our ω–shifting procedure.

Once the growth sets in, the issue is to establish the value of ω_s appearing in (9). This is a conceptually complex question because in contrast to the fixed-coupling case, ω_s no longer corresponds to a Regge singularity.

There are running-coupling diffusion corrections $\Delta(\alpha_s, Y)$ [30, 31, 32, 33], whose leading contribution, $\sim Y^3$,

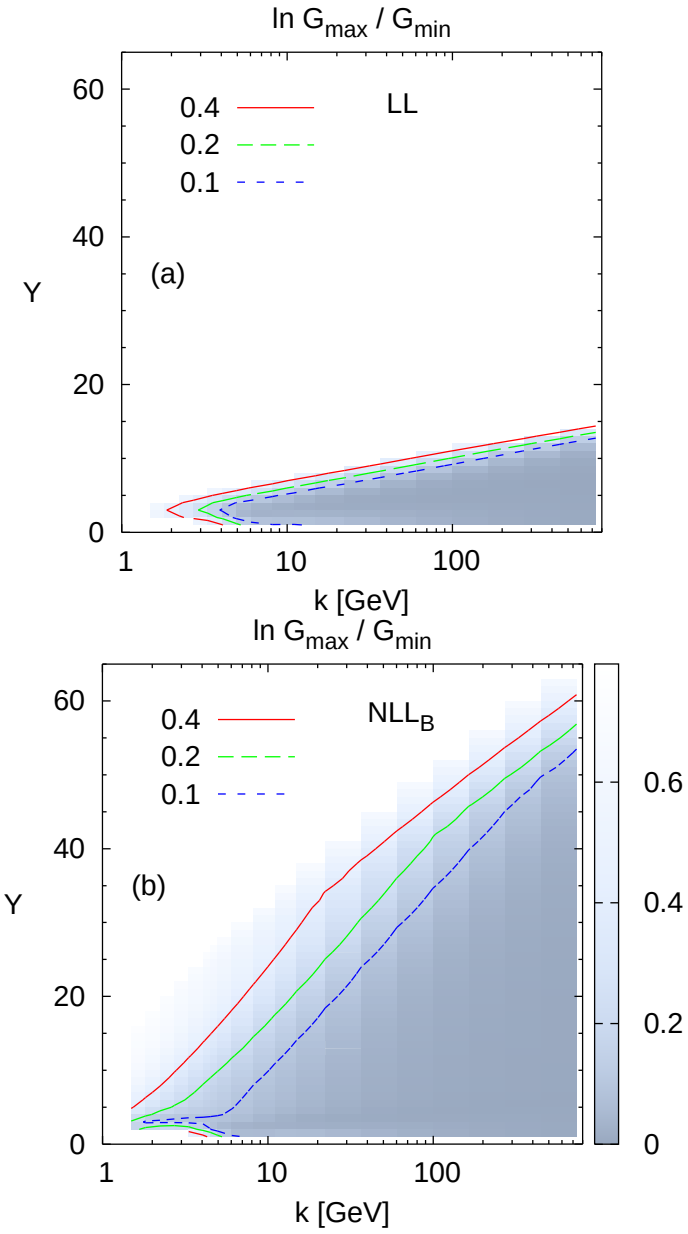

Figure 2. *Contour plots showing the sensitivity of $G(Y, k+\varepsilon, k-\varepsilon)$ to one's choice of non-perturbative regularisation, as obtained by examining the logarithm of the ratio of the regularisations giving the largest and smallest result for G. Darker shades indicate insensitivity to the NP regularisation, and contours have been drawn where the logarithm of the ratio is equal to* 0.1, 0.2 *and* 0.4. *Plot (a) shows the result for LL evolution, while (b) shows RGI NLL evolution (*$\mathrm{NLL_B}$*). The regularisations considered are those of Fig. 1.*

for this model is [20]

$$\Delta(\alpha_s, Y) \simeq \frac{Y^3}{24}\left[\frac{\partial}{\partial \log k^2}\omega_s(\alpha_s(k^2))\right]^2 \chi''_{\mathrm{eff}}(\tfrac{1}{2}). \qquad (12)$$

In addition, $\Delta(\alpha_s, Y)$ contains terms with weaker Y dependences, including Y^2 and Y. Such terms can be disen-

[3] Renormalisation scale uncertainties of the resummed results are sizeable – of the order of several tens of percent for $Y > 4$ – but seem anyhow quite modest compared to the order(s) of magnitude difference with LL. The x_μ dependence of the LL solution is somewhat smaller than for $\mathrm{NLL_B}$– this may seem surprising, but at larger Y the LL solution is in the NP domain, where non-linearities (in α_s) reduce the x_μ dependence.

tangled by the method of the b-expansion [38].[4]. Since

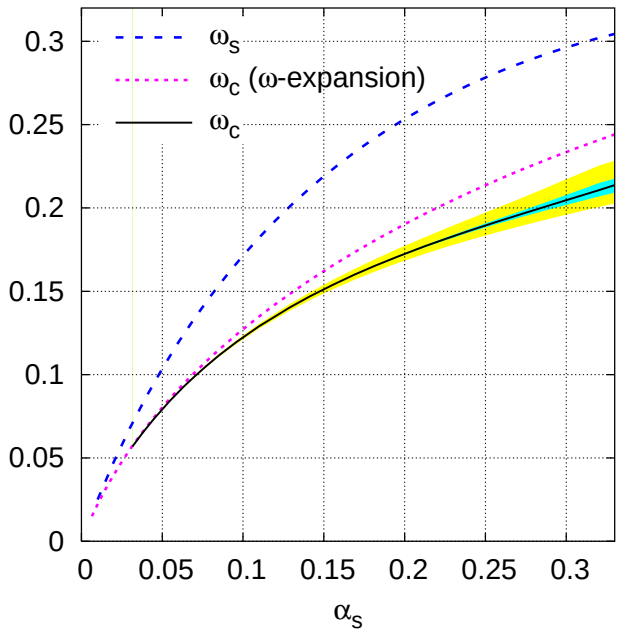

Figure 3. *The small-x exponents: the Green's function effective exponent ω_s is shown to first order in the b-expansion; the splitting function exponent ω_c is shown together with NP and renormalisation scale uncertainty bands, defined in figure 4. Also shown, for reference, is the result for ω_c using the method of [7], for $b(n_f = 4)$.*

running-coupling diffusion corrections start only at order b^2, it is possible to give an unambiguous definition of ω_s up to first order in b, while retaining all orders in α_s for non running-coupling effects. The result is shown in Fig. 3 as a function of α_s and as has been found in previous work [7], there is a sizeable decrease with respect to LL expectations.[5] Furthermore, the leading diffusion corrections in (12) turn out to be numerically small, about an order of magnitude down with respect to the LL result, due to a sizeable decrease of the diffusion coefficient $\chi''_{\rm eff}$, over and above the decrease already discussed for ω_s.

4 Resummed Splitting Function

The Green's function $G(Y;k,k_0)$ has been investigated in the collinear limit $k \gg k_0 \sim k$ also. In such a case the sensitivity to the IR regularization is much stronger. However, many arguments in the BFKL framework, [17, 16, 19, 7, 9, 29], have been given in favour of *factorisation*, Eq. 8, with the small-x splitting function $P_{gg}(z,Q^2)$ being independent of the IR regularisation. The most dramatic demonstration of factorisation is perhaps in the fact that a numerical extraction of the splitting function from the Green's function by deconvolution gives almost identical splitting functions regardless of the regularisation. This is illustrated in Fig. 4, where the solid line and its inner band represent the result of the deconvolution together with the uncertainty resulting from the differences between the three cutoff regularisations. The resulting regularisation dependence is pretty small, and at higher Q it diminishes rapidly as an inverse power of Q, as expected from a higher twist effect.

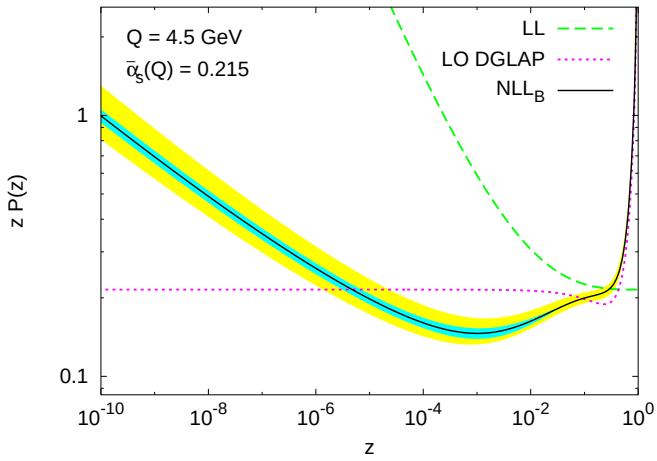

Figure 4. *Small-x* $\mathrm{NLL_B}$ *resummed splitting function, compared to the pure 1-loop DGLAP and the (fixed-coupling) LL BFKL splitting functions. The central* $\mathrm{NLL_B}$ *result corresponds to* $x_\mu = 1$, $\bar{k} = 0.74$ GeV*; the inner band is that obtained by varying* $\bar{k}$ *between* 0.5 *and* 1.0 GeV*, while the outer band corresponds to* $\frac{1}{2} < x_\mu^2 < 2$.

Several features of the resummed splitting function are worth commenting and comparing with previous NLL calculations, including various types of resummations [7, 9, 10]. Firstly, at very large-x it approaches the normal DGLAP splitting function. The momentum sum rule is satisfied to within a few parts in 10^4. At moderately small-x the splitting function is quite strongly suppressed with respect to the LL result and shows not a power growth, but instead a significant dip (of about 30% relative to the LO DGLAP value, for $\bar{\alpha}_s = 0.215$). Dips of various sizes and positions have been observed before in [10, 8, 29] though this is significantly shallower than that found in [10] at NLL order and similar to that found in the ω-expansion [20] and in the duality approach [9, 22].

At very small-x one finally sees the BFKL growth of the splitting function. We recall that the branch cut, present for a fixed coupling, gets broken up into a string of poles,

[4] In the $b \to 0$ limit, with $\alpha_s(k^2)$ kept almost fixed, the non-perturbative Pomeron is exponentially suppressed [38, 9], so that the b-expansion can also be used as a way of defining a purely perturbative Green's function without recourse to any particular infrared regularisation of the coupling [38].

[5] A direct comparison with earlier results for ω_s [7] is not possible, because they are based on a different definition (the saddle-point of an effective characteristic function), which is less directly related to the Green's function. Nevertheless, the present results are consistent with previous ones to within NNLL uncertainties.

with the rightmost pole located at ω_c, to the left of the original branch point (ω_s), $\omega_s - \omega_c \sim b^{2/3}\alpha_s^{5/3}$ [7]. The origin of this correction is similar to that of the $b^{2/3}\alpha_s^{5/3}$ contributions to ω_P for cutoff regularisations [39]. The dependence of ω_c on Q is shown in Fig. 3 together with its scale and IR regularisation dependence. It is slightly lower than the earlier determination in the ω-expansion [7][6]. Both determinations are substantially below ω_s, as expected.

To sum up, the resummed gluon density at comparable scales $k \sim k_0$ stays perturbative for a wide rapidity range - including most of the phase space available at next generation colliders - due to a suppression of the non-perturbative Pomeron and of diffusion corrections (Fig.2). The expected increase with energy starts slowly (Fig.1), due to small-x phase space effects, and is regulated by the exponent ω_s (Fig. 3) which, in the relevant Q-range is roughly of size $\alpha_s(Q^2)$. In the collinear region $k \gg k_0$ the gluon splitting function can be factored out, shows a shallow dip in the moderate small-x range (Fig.4), and its increase is regulated by the exponent $\omega_c < \omega_s$, also shown in Fig. 3.

A realistic prediction of cross-sections requires the inclusion of impact factors along the lines of [20], and of the quark sector along the lines of [19, 7]. But we can already say that resummed results show interesting preasymptotic effects and are, very roughly, closer to low order predictions than expected. This in turn may provide a preliminary explanation of the apparent smoothness of small-x cross-sections despite the occurrence, in their description, of large perturbative coefficients and of various strong-coupling phenomena.

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[6] when compared with the same flavour treatment — the value of ω_c in Fig. 6 of [7] actually refers to $b(n_f = 0)$, while that of Fig. 3 here is for $b(n_f = 4)$.

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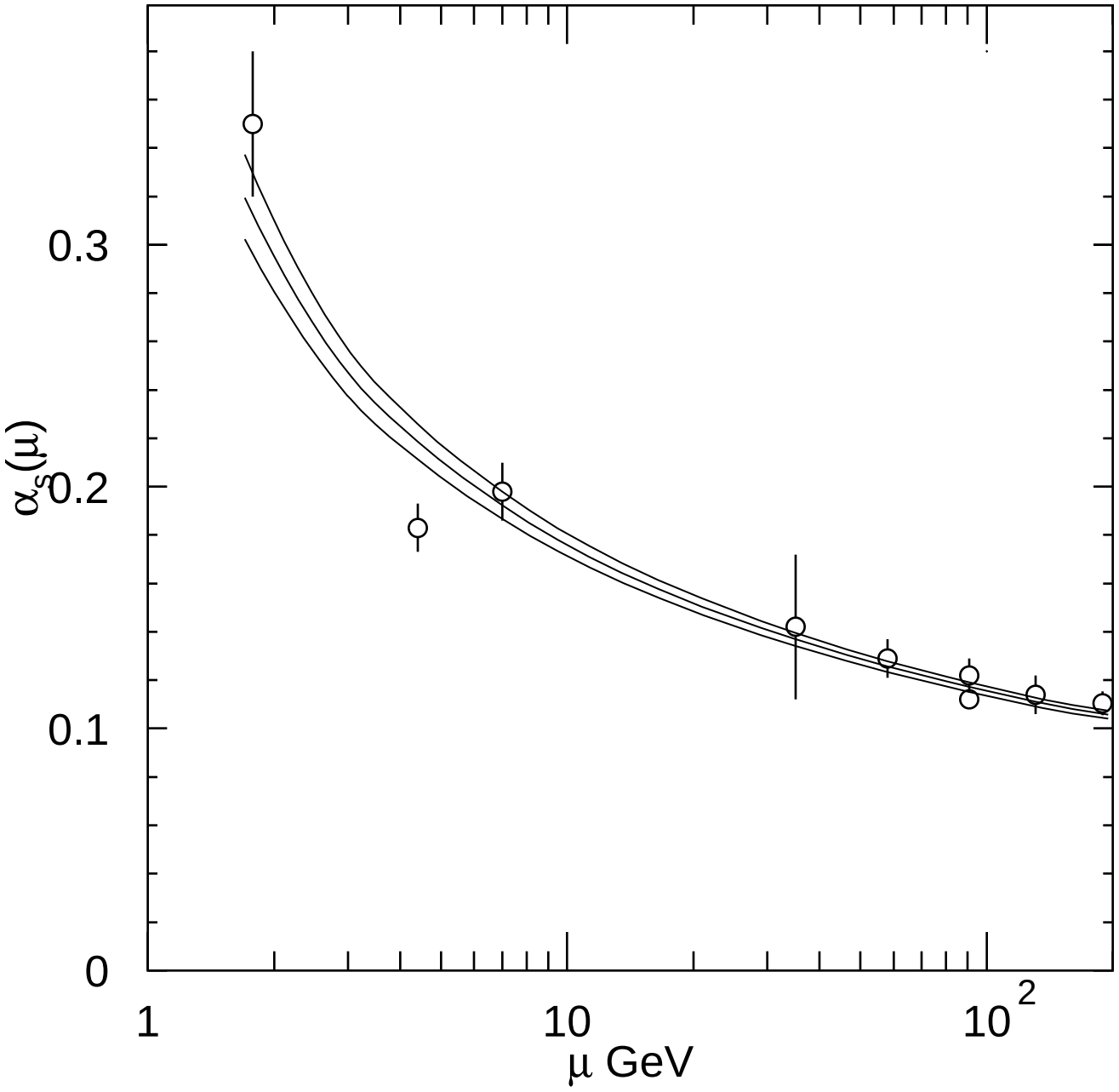
0.3
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$\alpha_s(\mu)$
1
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μ GeV